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Guido Fioretti

University of Bologna

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# Two Lyapunov Functions for Flexible Organizations

Guido Fioretti  
University of Bologna  
guido.fioretti@unibo.it

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## Abstract

Since the second half of the past century, increasingly flexible organizational forms are appearing among firms. However, while hierarchies are easily described, too few mathematical tools are available for flexible organizations.

In this article, two Lyapunov functions are proposed in order to assess the state and trend of flexible organizations. The first of these functions is based on information waste. The second function is based on duplication of operations. The underlying idea is that firms tend towards organizational configurations where waste of information and duplication of operations are minimized.

**JEL:** L20, D29

**Keywords:** Flexible structures, New organizational forms

## 1 Introduction

Since several decades, many firms are shifting towards structures that are ever more flexible, decentralized, reticular or otherwise defined but, in some sense, different from classical hierarchies [20]. Possibly, even the classical shift from functional to multidivisional structures may be interpreted as a part of this secular trend [5]. Wherever its origin may be dated back, this process clearly accelerated since the 1980s, giving rise to a series of buzzwords such as “kanban system”, “teamwork”, “production islands”, “flat structures”, “network firm” and others more or less directly related to the idea of increasing flexibility.

Henceforth, the expression “flexible organization” will be used to capture organizational forms that, under some respect, depart from bureaucracies. This article takes the stand that, beyond the many managerial fads and myths surrounding flexibility and flexible organizations, a real trend towards flexibility does exist.

According to many observers, this trend is mainly due to increasing environmental unpredictability induced by increasing variety of tastes and products [14]. Indeed firms operating in predictable environments, such as some large companies operating in the

mining sector that never switched to the multidivisional structure, are escaping the general trend [9].

In any case, increasing flexibility should not be seen as a compelling trend for *all* firms. It is clear that, although some firms are developing highly decentralized and flexible forms, others just softened the bureaucratic procedures that they had adopted in the 1960s and 1970s, while still others are by no means affected by the rush towards flexibility. One may conclude that albeit a trend does exist, this does not imply a radical transformation of all firms but rather the emergence of a novel organizational form that will possibly coexist with the previous ones.

This novel organizational form rejects a classical prescription of organization theory, namely, that the productive core of a firm should be isolated from all unpredictable environmental variability [36]. Rather than buffering disturbances by means of inventories and standardized procedures, these firms accept that unpredictable signals enter their daily operations in order to exploit market niches — for instance, a firm may care for customizations and dedicated production lots in order to serve particular needs.

It is obvious that a firm that sets out to exploit an unpredictable environment must entail a wider array of behavioral patterns than a firm designed to cope with a stable and predictable environment. Furthermore, Ashby's principle of requisite variety may suggest that the array of behavioral patterns of such a firm should be even larger than that of the environment itself [3].

These quite general considerations suggest that firms characterized by a flexible organization should exhibit both a high cognitive ability, in order to understand novel situations, and a substantial innovative ability, in order to cope with novel situations. These two features may be expressed as follows:

1. The firm must be able to understand and adapt to novel situations. Therefore, it must be able to classify a huge amount of information into categories, adapt its categories to the changing environment and develop proper patterns of action. Since classification implies that slightly different pieces of information are considered as equivalent to one another, information classification implies that some information is wasted in the process.
2. The firm must be able to explore novel arrangements of its assets in order to satisfy novel needs. Thus, it must be able to try novel combinations and sequencings of its operations. Since by doing so it may occur that different parts of the firm explore the same combination and sequencing of operations, some unnecessary duplication of operations may take place.

The above issues regard (1) a firm's ability to recognize the relevant features of an unpredictable environment, and (2) its ability to explore appropriate reactions. Thus, they constitute sensible and valuable characteristics.

However, both of them have drawbacks. In fact, with respect to a firm operating in a predictable environment it appears that a firm capable of recognizing novelties and developing appropriate responses may (1) waste too much information, and (2) duplicate too many operations.

Thus, under these respects the firms operating in unpredictable environments are less efficient than firms operating in predictable environments. Provided that firms

seek efficiency, it follows that firms have a tendency to reduce information waste and duplication of operations, though the unpredictability of their environment may prevent them from reaching a state where no information is wasted and no operation is duplicated.

Thus, the extent of information waste and the extent of operations duplication may play for the study of organizational forms the same role played by utility and profit functions in economics [8], fitness functions in evolutionary theory, gravitational and electrical potential functions in physics. All of them are Lyapunov functions (see Appendix A) describing the tendency of a system towards a stable equilibrium state.

Lyapunov functions describe a tendency that is not necessarily followed, but that nevertheless exerts a pressure pointing to a stable equilibrium. So microeconomics depicts firms as profit maximizers, evolutionary theory depicts species as fitness maximizers and physics describes falling bodies as potential minimizers; however, firms may not reach maximum profit because of management inefficiencies, exogenous constraints or other reasons, species may not reach maximum fitness because some evolutionary jumps may be physically impossible, and some falling bodies may be impaired from falling to the soil by forces acting in the opposite direction. The existence of a Lyapunov function does not imply that the stable equilibrium will necessarily be reached before the system undergoes a transformation that changes the equilibrium itself. It just ensures that a tendency towards that equilibrium exists.

In this article, information waste and operations duplication are proposed as Lyapunov functions to characterize flexible organizations. The underlying idea is that, since wasting information and duplicating operations are not good things by themselves, a tendency exists in all organizations to minimize them. However, the stable equilibrium where no information is wasted and no operation is duplicated can only be reached if the environment is perfectly predictable. Only if its environment is perfectly predictable a firm can afford not to be flexible at all. In general, the less predictable the environment, the more information is wasted and the more operations are duplicated. In other words, the less predictable the environment, the more flexible the organization.

If these two Lyapunov functions provide a valuable synthetic description of the distance of flexible organizational forms from the ideal organization of a firm operating in a predictable environment, then they can be used to compare alternative organizational forms and to evaluate their performance across environments. Similarly to comparisons of levels of utility and levels of profits, they may enable a comparative statics of organizational forms.

The ensuing section 2 defines and expounds the two Lyapunov functions. Section 3 sketches their possible application to numerical and empirical examples. Finally, section 4 frames this formalism with respect to the theories of the firm.

## **2 Two Lyapunov Functions**

Let us conceive a firm as composed by organizational units that may entail both men and machines. Let us assume that each organizational unit is endowed with (1) a set of categories to classify information, (2) a set of pieces of information to be produced, and (3) a rule that specifies on what occasions a particular piece of information should

#	0	0	1	1
1	1	1	1	1
#	0	1	0	1
0	0	0	0	0

Figure 1: A category with two “don’t care” characters and the four information strings that it classifies.

be produced.

The categories represent the situations that decision-makers endowed with particular machines are able to recognize. For instance, a worker at a press may recognize any metal sheet as pertaining to him, whereas he would refuse paperwork as pertaining to somebody else.

The rule specifies which operation an organizational unit should carry out when certain situations are recognized. For instance our worker at the press, after recognizing a metal sheet may put it under the press, activate it and obtain an object with the desired shape.

The output of an organizational unit may be a physical object but also a piece of software or a communication. In any case, it is perceived by the receiving unit as a piece of information. For instance, for the worker at the press in the above example the metal sheet entails the information “this sheet must be shaped”.

The above description of organizational units is quite general. The operations carried out by an organizational unit consist of producing as output a piece of information when a piece of information is received as input. The input information is classified by a category and the decision concerning which information should be issued as output is made by means of a rule.

Taking inspiration from classifier systems [11] [12], let us represent both categories and information by means of strings of characters that can be zeros, ones, or “don’t care” characters #.<sup>1</sup> Henceforth we shall also employ the terms “category string” and “information string”, respectively.

Category strings must entail at least one #-character. In fact, categories are strings that match all information strings that have zeros and ones in the same positions where they have zeros and ones, while it does not matter which character information strings have where category strings have a #. For instance, the category string depicted in the left half of figure 1 is able to classify the four information strings depicted in the right half of the picture. One may also visualize a category string as a container that collects all information strings that have zeros and ones in its same positions.

Information strings may have #-characters as well. In this case, #-characters represent the unpredictability of the corresponding information bit. Thus, if an information

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<sup>1</sup>Classifier systems employ the terms “condition” and “action” where we said “category” and “information”, respectively.

string with a # is issued, this can only be classified by a category string that has a # in the same position.

Let  $H$  denote the number of different category strings owned by all organizational units of the firm. Let  $K$  denote the number of different information strings that can be produced by all organizational units of the firm. Since categories are there in order to classify information, it must be  $H < K$ .

Organizational units may be so simple to have just one category string and be able to produce just one kind of category string. This may be e.g. the case of a specialized machine, a worker on the conveyor belt, or a department in a strongly bureaucratic organization. In general, an organizational unit may own several categories, each of a different kind, that enable it to process different information strings. This may be the case of a machine that can be endowed with different tools, a worker with multiple skills, or a problem-solving unit where different specialties are represented.

Connections between any two organizational units take place with varying frequency depending on hierarchical relations, time required to carry out operations, relatedness of activities, patterns of acquaintance or else. If we assume that the connections between any two organizational units only depend on what category strings they own and what information strings they produce, then the structure of relationships within the firm can be expressed by the probabilities that a category string of type  $h$  classifies an information string of type  $k$ ,  $\forall h = 1, 2, \dots, H$  and  $\forall k = 1, 2, \dots, K$ . Let  $p_{hk}$  denote the probability that a category string of type  $h$  classifies an information string of type  $k$ .

The existence of a Lyapunov function implies that the operations carried out by the units and the probabilities  $p_{hk}$  have a tendency towards an equilibrium configuration. If the Lyapunov function rests on structural properties, this equilibrium state is structurally optimal.

A firm can attain structural optimality if it finds itself in a predictable environment and if it is capable of making predictions. In order to characterize the equilibrium state where no information is wasted and no operations are duplicated, let us introduce the following definitions:

**Definition 1** The environment of a firm is *predictable* if it behaves according to laws that are known by the firm, and if these laws do not require unavailable data in order to be applied.

**Definition 2** Within a firm, a decision-maker is *infallible* if she makes decisions that do not impair the attainment of its goals.

Clearly, infallible decision-makers in a predictable environment are an abstraction. They are made in order to claim that, if the environment is predictable and the decision-makers are infallible, then a firm tends to a stable equilibrium state where no information is wasted and no operations are duplicated.

The current trend towards flexible organizations is generally interpreted as due to the fact that the environments where firms operate become ever less predictable with respect to managerial capabilities. Coherence with this interpretation implies that the natural tendency acts in the opposite direction, i.e. towards rigid, perfectly planned organizations where no information is wasted and no operations are duplicated. If the

opposite trend is observed, this can only be due to increasingly unpredictable environments coupled with the bounded rationality of decision-makers.

Consider the classical case of Toyota, which in the 1980s was able to develop new models faster than any American or Japanese competitor because within this firm (i) technical specifications fluctuated within broad ranges that were narrowed as late as possible (broad categories that waste a large amount of information), and (ii) a large number of alternative projects was carried out at the same time (duplication of operations) [38]. If the success of Toyota is ascribed to increased environmental unpredictability, this implicitly means that its U.S. competitors had actually achieved a higher degree of structural optimality with respect to an environment that was supposed to be stable but, alas, started to change in unpredictable ways. The fact that at Toyota technical specifications fluctuated and several alternative projects were carried out at the same time is not good in itself, though it enabled greater flexibility. In a more predictable environment, as those prevailing in the 1960s, this flexibility had been useless.

In the following two subsections, two Lyapunov functions will be introduced, that describe the natural tendency of an organization to the state where no information is wasted and no operation is duplicated. For the idea of defining potential functions to describe the evolution of a decentralized economic system, Allais [1] has to be credited.

## 2.1 Information Waste

Let *specificity* denote the number of non-# characters in a string. Let  $s_h$  denote the specificity of category string  $h$ , where  $h = 1, 2, \dots, H$ , and let  $z_k$  denote the specificity of information string  $k$ , where  $k = 1, 2, \dots, K$ .

Let us define the *information waste* when the category string  $h$  classifies the information string  $k$  as follows:

$$u_{hk} = z_k - s_h \quad (1)$$

Note that since  $z_k \geq s_h$ , it follows that  $u_{hk} \geq 0$ .

The waste of information in the whole firm is the sum of the information wasted at all connections. However, since connections occur with specific probabilities, the addends of this sum must be weighted:

$$U = \sum_{hk} p_{hk} u_{hk} \quad (2)$$

If a firm starts with a structure  $S_*$  and arrives at a structure  $S^*$ , the corresponding variation of information waste is  $\Delta U$ :

$$\Delta U = \sum_{S_i=S_*}^{S^*} \Delta U_i \quad (3)$$

where  $\Delta U_i = U|_{S_i} - U|_{S_{i-1}}$  and where the series of structures  $\{S_i\}$  extends from  $S_*$  to  $S^*$ .

If  $\Delta U_i < 0$  for  $\forall i$ , then the organization has a spontaneous tendency to move from structure  $S_*$  to structure  $S^*$ .

## 2.2 Operations Duplication

The operations carried out by an organizational unit are the procedures it follows in order to produce an information string out of an input classified by one of its category strings. Thus, in order to consider the operations carried out by an organization unit in terms of the probability that its category string captures an information string and that its information string is captured by a category string, we must consider chains of at least three organizational units.

Let us consider operations duplication by organizational units that classify information by means of category strings of type  $h_1$  and produce information strings of type  $k_1$ . Let us consider chains of three organizational units that include the above units in the middle. Let the stream of information be  $k \rightarrow h_1 \mapsto k_1 \rightarrow h$ , where  $k$  is the information string produced by the first unit and  $h$  is the category string employed by the third unit, the symbol “ $\rightarrow$ ” denotes a connection between two organizational units whilst “ $\mapsto$ ” denotes the operations carried out by the central unit in the chain.

Duplication of the operations carried out by one organizational unit takes place if two chains of three organizational units are linked by identical category strings and identical information strings. The probability that two chains of three units duplicate the operations of their central unit is:

$$v_1^2 = \frac{1}{HK} \sum_{h,h_1=1}^H \sum_{k,k_1=1}^K (p_{hk_1} p_{h_1k})^2 \quad (4)$$

where coefficient  $1/HK$  ensures that  $0 \leq v_1^2 \leq 1$ .

Likewise, the probability that three chains of three units triplicate the operations of their central unit is:

$$v_1^3 = \frac{1}{HK} \sum_{h,h_1=1}^H \sum_{k,k_1=1}^K (p_{hk_1} p_{h_1k})^3 \quad (5)$$

These terms entail a geometric series of ratio  $\sum_{h,h_1=1}^H \sum_{k,k_1=1}^K (p_{hk_1} p_{h_1k})$ . So if in the organization there are  $m$  units that duplicate operations within chains of three units, the sum of this series is:

$$v_1(m) = \frac{1}{HK} \sum_{h,h_1=1}^H \sum_{k,k_1=1}^K (p_{hk_1} p_{h_1k})^2 \frac{1 - \left( \sum_{h,h_1=1}^H \sum_{k,k_1=1}^K p_{hk_1} p_{h_1k} \right)^m}{1 - \sum_{h,h_1=1}^H \sum_{k,k_1=1}^K p_{hk_1} p_{h_1k}} \quad (6)$$

provided that  $\sum_{h,h_1=1}^H \sum_{k,k_1=1}^K (p_{hk_1} p_{h_1k}) \neq 1$ .

If  $\sum_{h,h_1=1}^H \sum_{k,k_1=1}^K (p_{hk_1} p_{h_1k}) < 1$  and if  $m$  is sufficiently large, the above value can be approximated by its asymptotic limit for  $m \rightarrow \infty$ :

$$v_1 = \frac{1}{HK} \frac{\sum_{h,h_1=1}^H \sum_{k,k_1=1}^K (p_{hk_1} p_{h_1k})^2}{1 - \sum_{h,h_1=1}^H \sum_{k,k_1=1}^K p_{hk_1} p_{h_1k}} \quad (7)$$

The above expressions refer to chains of three organizational units where the operations carried out by the central unit are duplicated, triplicated, and so on. We may



consider duplication of the operations carried out by a chain of two organizational units, included in a chain of four units. In this case, the stream of information would be  $k \rightarrow h_1 \mapsto k_1 \rightarrow h_2 \mapsto k_2 \rightarrow h$ , where  $k$  is the information string produced by the first unit,  $h_1$  and  $k_1$  are, respectively, the category string and the information string of the second unit,  $h_2$  and  $k_2$  are, respectively, the category string and the information string of the third unit, and  $h$  is the category string employed by the fourth and last unit.

The probability that two or more chains of four organizational units duplicate operations is:

$$v_2^2 = \frac{1}{HK} \sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K (p_{hk_1} p_{h_1 k_2} p_{h_2 k})^2 \quad (8)$$

where, again, coefficient  $1/HK$  ensures that  $0 \leq v_2^2 \leq 1$ .

As above, we may consider triplication, quadruplication, and so on. The ensuing series is geometric of ratio  $\sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K p_{hk_1} p_{h_1 k_2} p_{h_2 k}$  and, as above, its sum to the  $m$ -th order is:

$$v_2(m) = \frac{1}{HK} \sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K (p_{hk_1} p_{h_1 k_2} p_{h_2 k})^2 \frac{1 - \left( \sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K p_{hk_1} p_{h_1 k_2} p_{h_2 k} \right)^m}{1 - \sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K p_{hk_1} p_{h_1 k_2} p_{h_2 k}} \quad (9)$$

provided that  $\sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K p_{hk_1} p_{h_1 k_2} p_{h_2 k} \neq 1$ .

As above, the limit of this sum for  $m \rightarrow \infty$  is:

$$v_2 = \frac{1}{HK} \frac{\sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K (p_{hk_1} p_{h_1 k_2} p_{h_2 k})^2}{1 - \sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K p_{hk_1} p_{h_1 k_2} p_{h_2 k}} \quad (10)$$

provided that  $\sum_{h,h_1,h_2=1}^H \sum_{k,k_1,k_2=1}^K p_{hk_1} p_{h_1 k_2} p_{h_2 k} < 1$ .

The more units are involved, the less likely it is that duplication of operations occurs. Thus  $\{v_1, v_2, \dots\}$  is a decreasing series, whose generic  $n$ -th term takes the form:

$$v_n = \frac{1}{HK} \frac{\sum_{h,h_1,\dots,h_n=1}^H \sum_{k,k_1,\dots,k_n=1}^K (p_{hk_1} p_{h_1 k_2} \dots p_{h_{n-1} k_n} p_{h_n k})^2}{\sum_{h,h_1,\dots,h_n=1}^H \sum_{k,k_1,\dots,k_n=1}^K p_{hk_1} p_{h_1 k_2} \dots p_{h_{n-1} k_n} p_{h_n k}} \quad (11)$$

In the end, in a firm with multiple paths up to order  $N$  we can measure the extent of operations duplication by means of:

$$V = \sum_{n=1}^N v_n \quad (12)$$

Similarly to equation (3), we can say that if a firm starts with a structure  $S_*$  and arrives at a structure  $S^*$ , the corresponding variation of operations duplication is  $\Delta V$ :

$$\Delta V = \sum_{S_i=S_*}^{S^*} \Delta V_i \quad (13)$$

where  $\Delta V_i = V|_{S_i} - V|_{S_{i-1}}$  and where the series of structures  $\{S_i\}$  extends from  $S_*$  to  $S^*$ .

If  $\Delta V_i < 0$  for  $\forall i$ , then the organization has a spontaneous tendency to move from structure  $S_*$  to structure  $S^*$ .

### 3 Applications

This sections illustrates the meaning of the Lyapunov functions defined above by means of two examples. The first one concerns a stylized comparison between a flexible organization and a hierarchy according to quite a common scheme in the literature. The second one is excerpted from a real case of a firm that, after switching from a classical hierarchy to an extreme form of flexible organization, decreased its level of flexibility at a later stage. Lack of empirical information makes it impossible to apply the equations derived above, but the situation is suggestive of their meaning.

#### 3.1 Hierarchy vs. Multihierarchy

A number of comparisons between hierarchies and flexible organizations were prompted by the superior performance of many Japanese firms with respect to their American competitors during the 1980s. Soon, this became a canonical topic in organization science. Western investigators pointed to communication between marketing people and engineers, personnel rotation and flexible teams composed by people from different departments as key factors of the success of Japanese firms.<sup>2</sup>

Following this interpretation, Japanese-style firms have been characterized as *pol-yarchies*, or *multihierarchies*, i.e. structures where cross-connections between departments originate from superimposition of multiple hierarchies [33] [34] [35] [2]. Figure 2 illustrates the difference between hierarchies and multihierarchies in a highly simplified setting. Both structures entail a commanding unit (a) (the boss) and two functional units (b) and (c) (e.g. engineering and marketing) that process information stemming from two sources (d) and (e) (e.g. technological constraints and market surveys). Contrary to the simple hierarchy, the multihierarchy forces engineers to take account of aesthetical aspects and marketeers to take account of technical aspects.

Let us compute potential functions  $U$  and  $V$  for the hierarchy and the multihierarchy of figure 2. If a firm is endowed with infallible managers and if it operates in a predictable environment, it should move to the structure that has lower  $U$  and lower  $V$ .

Since in the hierarchy each organizational unit receives information from only one other unit and passes on information to only one other unit, each organizational unit has its specific category string and information string, which we may label with numbers following alphabetical ordering. So unit (a) has a category string labelled by  $h = 1$  and issues an information string labelled by  $k = 1$ , unit (b) has a category string labelled by  $h = 2$  and issues an information string labelled by  $k = 2$ , and so on.

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<sup>2</sup>Subsequent investigations yielded a much richer, often different picture of Japanese-style manufacturing. However, here the issue is that of comparing idealized structures that have been widely discussed in the literature.

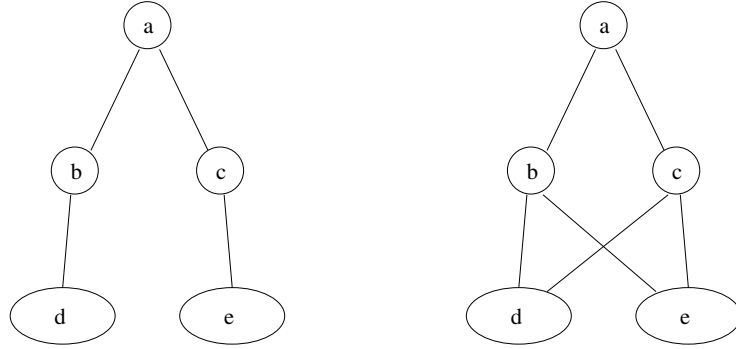


Figure 2: In the “Western” hierarchy on the left, units (b) and (c) rely on information issued by (d) and (e), respectively. In the “Japanese” multihierarchy on the right, units (b) and (c) rely on information issued by both (d) and (e).

On the contrary, in the multihierarchy units (b) and (c) must be endowed with similar category strings in order to be able to process both the information strings issued by (d) and the information strings issued by (e). One possibility is that (b) and (c) are endowed with one category each, but one with a large number of #-characters — i.e. that they become generalists. In this case units (b) and (c) are endowed with the same category strings and produce the same information strings, that are different from those they had in the hierarchy. Another possibility is that (b) and (c) are endowed with two categories each, i.e. that each of them acquires the skills of two specialties. In this case each unit has a different pair of category strings, but may produce the same information string as in the hierarchy.

Thus, we shall consider three organizational configurations: the hierarchy, the multihierarchy with generalists and the multihierarchy with multi-skilled specialists. Lyapunov functions  $U$  and  $V$  will be computed in all three cases, and comparisons will be made between the hierarchy and each of the two versions of the multihierarchy.

Table (1) illustrates the codification of category strings and information strings in the three configurations. In the hierarchy, each unit has a category string and issues an information string denoted by a number corresponding to the name of that unit. In the multihierarchy with generalists, units (b) and (c) have the same category string, denoted by  $h = 6$ , and produce the same information string, denoted by  $k = 6$ . For simplicity it is assumed that the category string of unit (a) is still able to capture the information strings issued by (b) and (c). In the multihierarchy with specialists, units (b) and (c) have two category strings each, denoted by  $h = 7, h = 8$  and  $h = 9, h = 10$ , respectively. However, they produce the same information strings as in the hierarchy.

Let us compute information waste  $U$ . The connection between (a) and (b) and the connection between (a) and (c) are characterized by the same amount of information waste in the hierarchy as in the multihierarchy with multi-skilled specialists. So far it regards the multihierarchy with generalists, if we for simplicity assume that  $z_6 = 1/2(z_2 + z_3)$  also in this case these connections contribute the same amount of information waste. Thus, under this assumption the information waste contributed by

unit	Hierarchy		Multihierarchy (g)		Multihierarchy (s)	
	h	k	h	k	h	k
a	1	1	1	1	1	1
b	2	2	<b>6</b>	<b>6</b>	<b>7, 8</b>	2
c	3	3	<b>6</b>	<b>6</b>	<b>9, 10</b>	3
d	4	4	4	4	4	4
e	5	5	5	5	5	5

Table 1: Codification of category strings and information strings for the hierarchy, the multihierarchy with generalists (g) and the multihierarchy with multi-skilled specialists (s). Codifications departing from those of the hierarchy have been highlighted.

the connections between (a), (b), (c) can be ignored.

By applying equation (3) we find that hierarchy, multihierarchy with generalists and multihierarchy with multi-skilled specialists are characterized by the following values of information waste:

$$U_H = p_{24}(z_4 - s_2) + p_{35}(z_5 - s_3) \quad (14)$$

$$U_{MHg} = 2p_{64}(z_4 - s_6) + 2p_{65}(z_5 - s_6) \quad (15)$$

$$U_{MHs} = p_{74}(z_4 - s_7) + p_{85}(z_5 - s_8) + p_{95}(z_5 - s_9) + p_{104}(z_4 - s_{10}) \quad (16)$$

Let us analyze the meaning of equations (14), (15) and (16). For simplicity, let us assume that  $p_{24} = p_{35} = p_{64} = p_{65} = p_{74} = p_{85} = p_{95} = p_{104}$  so probability values can be ignored altogether.

Let us consider the case of the multihierarchy with generalists. It is easy to see that  $U_{MHg} < U_H$ , i.e. that a firm has a tendency to switch from the hierarchical to the multihierarchical structure if  $s_6 > 1/4(s_2 + s_3 + z_4 + z_5)$ , i.e. if the category string of the two generalists in (b) and (c) is more specific than the average of the other strings considered. This is nearly impossible because by definition  $z_4 \geq s_2$  and  $z_5 \geq s_3$  and, furthermore, in order for  $h = 6$  to be a generalist's category it must be  $s_6 \leq \min\{s_2, s_3\}$ .

Thus, we must conclude that no spontaneous tendency exists for a firm to switch from a hierarchical structure to a multihierarchical structure implemented by generalists. This does not mean that such a shift may not occur if market unpredictability forces a firm to do so, but that such a shift would take place at the expense of a greater information waste. For instance, if the structure of figure (2) is interpreted in the sense that unit (b) deals with engineering and unit (c) deals with marketing, implementing the multihierarchy by means of generalists means that marketing is so basic that even engineers can do it and that technology is so simple that even the marketing people can

understand it, which implies that the firm is unable to exploit the most sophisticated market niches. Although environmental turbulence or other exogenous factors may force a firm to do so, it makes sense that no spontaneous tendency exists for this move.

Let us consider the case of the multihierarchy with multi-skilled generalists. In this case  $U_{MHs} < U_H$ , which means that a firm has a tendency to switch from the hierarchical to the multihierarchical structure if  $s_7 + s_8 + s_9 + s_{10} > s_2 + s_3 + z_4 + z_5$  which, since by definition  $z_4 \geq s_2$  and  $z_5 \geq s_3$ , implies  $s_7 + s_8 + s_9 + s_{10} > 2(s_2 + s_3)$ . Thus, a spontaneous tendency towards a multihierarchy exists only if the category strings of the multihierarchy, though obeying the requirements  $s_7 \leq z_4$ ,  $s_7 \leq z_5$ ,  $s_8 \leq z_4$ ,  $s_8 \leq z_5$ ,  $s_9 \leq z_4$ ,  $s_9 \leq z_5$ ,  $s_{10} \leq z_4$ ,  $s_{10} \leq z_5$ , are on average more specific than the category strings of the hierarchy. If the structure of figure (2) is interpreted in the sense that unit (b) deals with engineering and unit (c) deals with marketing, implementing the multihierarchy by means of multi-skilled specialists means that the engineers acquire a profound competence in marketing and that the marketing people acquire a profound competence in engineering. Although theoretically feasible, in practice this is nearly impossible or, at the very least, extremely costly. However, if environmental turbulence or other exogenous factors push a firm towards a multihierarchical structure, enhancing the competences of its organizational units may limit its drawbacks so far it concerns information waste.

Let us consider duplication of operations as expressed by  $V$ . The hierarchy on the left side of figure 2 does not duplicate any operation. The multihierarchy on the right side of figure 2, if it is implemented with generalists at units (b) and (c), duplicates the operation within the path  $z_4 \rightarrow h_6 \mapsto k_6 \rightarrow h_1$  as well as the operation within the path  $z_5 \rightarrow h_6 \mapsto k_6 \rightarrow h_1$ . Thus, in this case  $V \equiv v_1^2$ . The multihierarchy with multi-skilled specialists does not duplicate any operation.

In the case of the hierarchy,  $H = K = 6$ . In the case of the multihierarchy with generalists,  $H = K = 4$ . In the case of the multihierarchy with multi-skilled specialists  $H = 7$  and  $K = 5$ . Equation (12) yields:

$$V_H = 0 \quad (17)$$

$$V_{MHg} = \frac{1}{16} (p_{64}^2 p_{16}^2 + p_{65}^2 p_{16}^2) \quad (18)$$

$$V_{MHs} = 0 \quad (19)$$

$$(20)$$

When comparing the hierarchy with the multihierarchy with generalists one observes that, since  $V_{MHg} > V_H$ , also from the point of view of operations duplication there exists no endogenous drive to move to a multihierarchy. Eventually, an endogenous drive exists in the opposite direction.

On the contrary, when comparing the hierarchy with the multihierarchy with multi-skilled specialists one observes that, since  $V_{MHs} = V_H$ , from the point of view of oper-

ations duplication a firm should be indifferent between these two structures. However, the multihierarchy is much more costly and difficult to implement than the hierarchy.

Again, also from the point of view of operations duplication we can interpret the empirically observed diffusion of multihierarchical structures as due to forces that are exogenous to the firm, whose negative effects a firm attempts to counterbalance by multi-skilling its employees. To the extent this is achieved, the drawbacks of a multihierarchy can be offset and a firm implementing this structure can be economically viable.

### 3.2 A Real Context

A real context may give an idea of the kind of situations where the above formalism may be applied. The following example is to be meant as a sketchy introduction to possible applications.

*Oticon* is a Danish producer of hearing aids that, after having been organized as a functional hierarchy until the end of the 1980s, implemented such a radical restructuring to be mentioned as a paramount example of a new emerging structure based on flexibility and teamwork [27] [32]. *Oticon*'s restructuring actually concerned only its headquarters, where R&D and strategic decisions were made; however, it was far more radical than many re-organizations of research departments, or innovative flexible organizations in research-based firms such as those operating in the bio-tech industry. Thus, the interest of the popular press is definitely justified.

In 1991, *Oticon*'s headquarters moved to a new location where open spaces substituted offices, informal communication substituted paperwork and hierarchy was reduced to two levels [22] [15]. Most importantly, employees were pruned to constitute work teams on whatever project they thought it was worth pursuing — mostly research projects, but there were also projects concerning internal organization as well as marketing projects. The project on which a team decided to work had to be formally approved by a committee; however, in practice, the committee approved any project and provided its leader with financial means to “hire” employees in the team. A sort of internal labor market was created. Employees were encouraged to work in several teams at the same time, in some teams as a member, in other teams as the leader. On average, each of the 150 employees worked in 3 – 4 teams. At any time, about 70 projects were active. Some projects lasted years, others were short-lived.

The reason for such a radical re-organizing was that, during the 1980s, *Oticon* had lost substantial market shares to competitors who were better able at innovating, notably exploiting digital technologies to build in-the-ear hearing aids. By allowing its researchers to pursue whatever avenue they might conceive, *Oticon* was able to produce a burst of innovations that quickly restored its market share [31] [37].

However, this extremely flexible structure did not last forever. The fact that any project was allowed to start implied that enormous amounts of resources were wasted, so after some time the CEO had to intervene to stop projects that were clearly unprofitable, a practice that was at odds with the declared management philosophy and which generated obvious discontent [7]. In 1996, *Oticon* re-organized itself into a structure where, albeit spontaneous formation of work teams was still possible, projects were closely scrutinized before approval, only few of them were actually funded and the

internal labor market was abolished. Since 1996 *Oticon* employees participate on average to 1 – 2 different teams instead of 3 – 4, which amounts to dropping the number of on-going projects from about 70 to about 30 <sup>3</sup>.

Whilst the 1991 re-organization was driven by market competition requiring greater innovative capability, the re-organization of 1996 was an internal affair suggested by the need to keep costs under control. Thus, *Oticon*'s story appears as a perfect example of the logic followed in this paper, namely, that environmental unpredictability may push organizations towards flexible structures but nevertheless, to the extent one focuses on endogenous factors, organizations have a natural tendency to move towards more classical configurations. In the light of this framework, *Oticon*'s restructuring of 1991 appears as a correct but excessive response to environmental requirements, which was subsequently reduced by the internal forces operating in the 1996 re-structuring.

The drawbacks of the extremely flexible organization adopted by *Oticon* from 1991 to 1995 can be easily understood in terms of information waste and duplication of operations. In fact, the empirical literature ascribes the misfunctionings of this period to the following factors [7]:

- Each employee was required to be proficient in several fields. In this way, fruitful interactions with colleagues with quite a different backgrounds would be eased. In the terms of our model, this translates into endowing organizational units with multiple category strings and, possibly, to ascribe them the ability to choose one out of several information strings to produce. However, this was apparently a minor problem in a high-tech firm such as *Oticon*, where all employees had a high degree of education.
- Since work teams competing with one another did not share information, they eventually duplicated research efforts. Evidently, this is an instance of duplication of operations. Furthermore, all work teams engaged in internal politics for approval by the CEO. This is also an instance of operations duplication, in the sense that lobbying activities were duplicated across teams. Apparently, duplication of operations was definitely the most important problem.

Unfortunately, no empirical study of the *Oticon* case recorded the birth, death and composition of work teams, the problems they were called to solve and the solutions they developed. However, we can speculate how a research could be carried out if data were available.

Each single employee may be considered an organizational unit. Work teams are temporary groupings of these units, wherein some operations are duplicated. For instance, if all teams engage in internal politics in order to influence the CEO, then a “political” sequence of operations is duplicated among all teams. However, sequences of technical operations may be duplicated as well, for instance if several teams start with the same idea (e.g. applying digital electronics to hearing aids), or just if a certain technical process is required by otherwise different projects.

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<sup>3</sup>This figure derives from my own calculations. The ratio of 3.5 (median number of participated teams from 1991 to 1996) to 1.5 (median number of participated teams since 1996) is 2.3. If this ratio reflects into the average number of work teams, they must have passed from 70 in 1991-1996 to 30 since 1996, on average.

If a careful mapping of *Oticon* had been recorded, the Lyapunov function (12) could be computed. Since this function yields lower values when few operations are duplicated, one may expect  $V$  to take a lower value after the 1996 restructuring. Thus, it indicates the direction of endogenous change.

## 4 Theoretical Framing

This article presented a technical tool to assess certain structural properties of a firm. Since no tool is neutral with respect to theory, it is important to conclude by framing it within the wider notion of the theory of the firm, which, as it stands, is split in two streams.

On the one hand, the contractual stream initiated by Coase [6] views the firm as arising because its transactions are more efficient than market transactions. This argument has been further deepened by Williamson [40] [41], who ascribed the origin of the inefficiency of market transactions to the possibility *opportunistic behavior*, which is difficult to detect because of human *bounded rationality*. March and Simon also wrote in the contractualist stream, though with a wider view [16]. According to March and Simon, the network of contracts that constitutes a firm originates because boundedly rational decision-makers are unable to supervise an environment that is inherently unpredictable, *also* — but not only — because of opportunistic behavior. By constituting a firm, the actors involved create a place where they enact their own (satisficing) thumb rules and cognitive maps of causes and effects.

On the other hand, the variably called “resource”, “competence”, “capability” or “knowledge” -based stream originated by Penrose [26] views a firm as an institution designed to develop and exploit specific competencies. Important developments within this stream included the evolutionary theory of the firm, based on the relative invariance of its routines [23], and the population ecology approach, which builds on routine invariance to apply the mutation-and-selection mechanism to populations of firms [10]. More recently, a cognitive stream emerged which understands “resources”, “competences”, “capabilities” or “knowledge” as deriving from psychic processes [39] [24] [25].

The cognitive trends in both the contractualist and the knowledge-based stream are interesting because they may build a bridge between them. Indeed, the difference between the cognitive version of the contractualist stream and the cognitive version of the knowledge-based stream are essentially a matter of looking at the firm bottom-up, i.e. starting from the cognitive processes of its members, or top-down, i.e. starting from the collective cognitive processes of the organization as a whole. However different these approaches may be, they are not inherently irreconcilable as the visions of Coase and Penrose appear to be.

The two Lyapunov functions presented in this article clearly concern the cognitive processes within a firm. In fact, the first one focuses on information categorization, which is the fundamental mechanism of cognition. The second one focuses on the exploration of novel sequences of operations, which is a fundamental activity of a firm that seeks to cope with an unpredictable environment. Indeed, the mathematical and computational models that have been developed within the above theoretical streams



can be easily understood within the framework of the Lyapunov functions presented herein.

The contractualist stream produced a series of models where the members of an organization are viewed as information processors. These models succeeded to derive alternative organizational structures from alternative configurations of costs of information processing, costs of information communication and costs of specialization [28] [30] [29] [4] [42] [43] [44]. All of these models can be thought as taking place at the point where no information is wasted and no operations are duplicated, i.e. the point where the equilibrium described by the Lyapunov functions defined in this article has been achieved.

The knowledge-based stream produced fewer mathematical and computational models on the interplay between cognition and organizational structures. However, it is interesting to remark that a series of models based on information categorization employs a formalism that is akin to that of the first of our Lyapunov functions [17] [18] [19]. In the case of the knowledge-based stream, its models work at a point that is distant from the equilibrium defined by our Lyapunov functions.

Thus, our Lyapunov functions suggest to view the contractualist approach as a special and particularly important case of the knowledge-based approach — the case that obtains when no information is wasted and no operations are duplicated. As one may expect given its cognitive focus, this interpretation contributes to bridge between the two streams of the theory of the firm.

A conciliatory attitude is definitely supported by the empirical evidence on the influence of organizational structure on efficiency, which suggests that although the knowledge-based approach has a larger explanatory power, both approaches are significant [13]. However, one should be aware that the consequences of a theoretical merger based on considering cognitive processes may be much deeper than merely bridging two theoretical streams. In fact, cognition may imply social values, and in this case, contracts may no longer depend solely on individuals' preferences [21].

## A Lyapunov Functions

A simple way to visualize stable equilibria is to think of a surface in a  $n + 1$ -dimensional space of  $n$  state variables, plus one variable for the height of this surface. Henceforth, this surface will be also called *landscape*. The  $n$ -dimensional space of the state variables will be called *state space*.

The projection of a point of this surface on the  $n$ -dimensional state space represents a state that the system can attain. Thus, the state of the system may be represented by the position of a ball on the landscape.

According to one possible convention, the lower points on the surface represent the most preferred states. Thus, a system has a tendency to move from higher points to lower points on the surface. Consequently, the bottom of valleys represent stable equilibrium points. Conversely, the peak of mountains represent unstable equilibrium points.

According to the opposite convention, the upper points on the surface represent the most preferred states. Thus, a system has a tendency to move from lower points

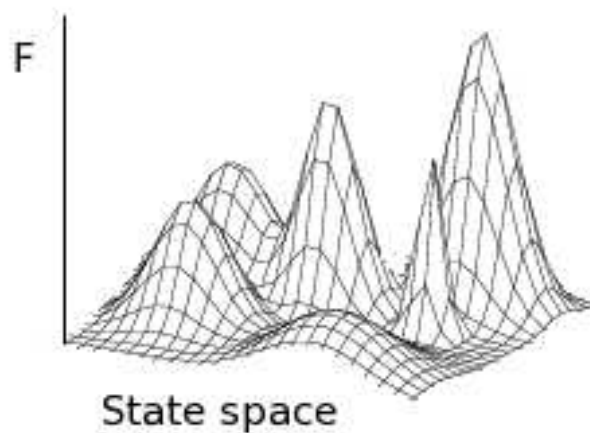


Figure 3: A surface representing equilibrium points on a two-dimensional state space. According to one convention, the bottom of valleys represent stable equilibrium points. According to the opposite convention, the top of mountains represent stable equilibrium points.

to upper points on the surface. Consequently, the top of mountains represent stable equilibrium points. Conversely, the bottom of valleys represent unstable equilibrium points.

Figure 3 illustrates one such surface in a case where  $n = 2$ . The state space is the  $X - Y$  plane.

In physics, this landscape is called *potential function*. Physics makes the convention that the lower points on this landscape represent the most preferred states. For instance, if this landscape represents the gravitational potential of the earth, this landscape coincides with the commonsense meaning of the word “landscape” with its mountains and valleys and a tendency for a body on top of a mountain to roll down to the bottom of the neighboring valley. Likewise, electrical potential describes the tendency of electrons to move from the negative to the positive pole.

In biology the potential function is called *fitness function*, or *fitness landscape*. Biology makes the opposite convention to physics: since organisms seek to improve their fitness, the higher points on the surface are the preferred states. So the ball representing a biological system has a tendency to climb mountains, mountain peaks are stable equilibria and the bottom of valleys are unstable equilibria.

Economics makes a similar point when it states that individuals seek to maximize utility: in this case, the landscape is the *social welfare function* obtained by aggregating individual utility functions. Also in this case, the effort of maximizing can be represented by climbing a peak.

The conventions employed in physics and respectively in biology and economics are mathematically equivalent to one another. In fact, minimizing a function  $F$  is equivalent to maximizing  $-F$ . Henceforth the convention will be used, that a function  $F$  has

to be minimized.

The potential functions of physics, fitness functions of biology and social welfare functions of economics are *Lyapunov functions*. The Lyapunov theorem states that, given a system described by state variables  $x_1, x_2, \dots, x_N$ , the origin of axes is a stable equilibrium point if:

1.  $\exists F(\mathbf{x}) \in C^o : F(0) = 0, F(\mathbf{x}) > 0$  around the origin;
2.  $\frac{\partial F}{\partial x_1} dx_1 + \frac{\partial F}{\partial x_2} dx_2 + \dots + \frac{\partial F}{\partial x_N} dx_N < 0$ .

The Lyapunov is expressed with respect to the origin of axes without any loss of generality. In fact, any point in the state space can be made the origin by means of a linear transformation.

The Lyapunov theorem says that, if we succeed to find a basin-shaped function such that the state of the system tends to move towards the lowest point, then that point is a stable equilibrium. Note that several Lyapunov functions may be defined for a system.

Note also that the Lyapunov theorem provides a sufficient, but not necessary stability criterium for an equilibrium point. Thus, if a Lyapunov function is found, we are certain that the equilibrium is stable. However, if no Lyapunov function is found we cannot conclude that an equilibrium point is unstable, and not even that it is or is not an equilibrium point.

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